In this assignment, we will design a positive column glow discharge as might be used in a lamp or “neon” sign. We have a discharge tube with dimensions as shown:

The cathode is a hollow cylindrical shell of length $L$ and diameter $D$. (Note: Consider the cathode as simply a cylindrical shell that is otherwise equivalent to a planar cathode. We are not talking about a current amplifying hollow-cathode discharge.) All of the useful light from the lamp comes from a long narrow tube housing the positive column. The tube that emits the light has diameter $d$ and length $L_c$. Using the ideal molecule cross sections with a Maxwellian electron energy distribution, find the:

1) Power of the light that is emitted (Watts) as a function of gas pressure (1-100 Torr).
2) Total voltage drop (cathode-to-anode) as a function of gas pressure (1-100 Torr).

Please plot and comment on the results. (Feel free to plot other quantities, such as $T_e$, $n_e$, $E/N$ if this helps to explain the results.) Use the following assumptions.

- Discharge is a normal glow.
- The glass tube is very close to the outer surface of the cathode, so current comes only from the inside surface of the cathode. Plasma fully covers the inner surface of the cathode.
- The inelastic processes for the ideal molecule are only excitation and ionization. (You do not need to re-derive the rate coefficients. Use the expressions from past homework assignments.)
- Every electron impact excitation results in the emission of a photon with wavelength 5000 Å. Light comes only from the positive column.
- The discharge consists only of the cathode dark space and positive column of length $L_c$. You can ignore any voltage drop across the negative glow.
- Electron loss in the positive column is dominated by ambipolar diffusion. (Note, at the higher pressures, this is a bad assumption, but we will use it. See question 3.)
- $L_c = 1.5$ meters, $D = 3$ cm, $d = 1$ cm, $L = 10$ cm.
- $T_{\text{gas}} = 400^\circ$K
- $\gamma$ (secondary electron emission coefficient by ion bombardment) = 0.15
- $M_{\text{gas}} = 28 \text{ AMU}$
- Ion mobility $\mu_I = \frac{1 \text{ cm}^2}{\text{V s}}$ at 1 atm (760 Torr)
- $\alpha$ (First Townsend Coefficient in cathode fall) = $A_p \exp(-B_p/E)$ cm$^{-1}$
  \[ A = \frac{10}{\text{cm} \text{ Torr}}, \quad B = \frac{200}{\text{cm} \text{ Torr}}, \quad p \text{ = gas pressure (Torr)}, \quad E = \text{Electric Field (V/cm)} \]

3) Please comment on how your results may change if we included electron-ion recombination as a loss process. (You do not need to do additional calculations – but please do discuss in some detail.)