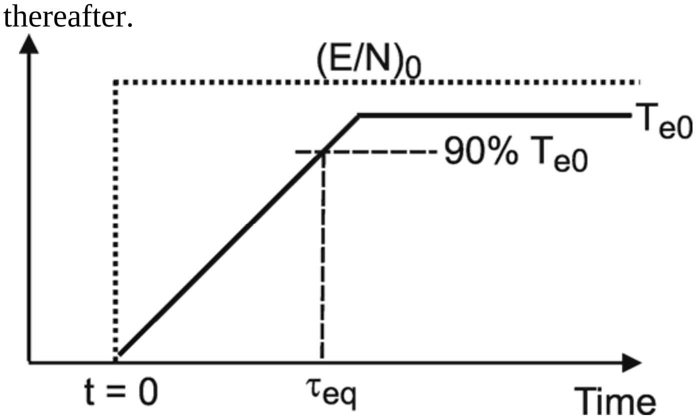
EECS 517 / NERS 578 FALL 2012 HOMEWORK #5

"Non-equilibrium" electron transport is important in low pressure electric discharge devices. "Equilibrium" transport usually refers to conditions where the local field approximation (LFA) is valid. The LFA states that, for a given gas mixture, the local instantaneous transport coefficients (e.g., electron temperature, ionization rate coefficients) can be uniquely specified by the value of the local instantaneous electric field (or E/N). This requires that the plasma is in the quasi-steady state [$dn/dt \approx 0$] and that spatial derivatives are not important [$dn/dx \approx 0$]. By quasi-steady state, we mean that the time required for the electron temperature to come into equilibrium with changes in the electric field is short compared to the time over which the electric field changes. Therefore, the electron temperature "tracks" changes in E/N. In "non-equilibrium" transport, the electron temperature, T_e, does not directly track changes in E/N. That is, the equilibration time for T_e is longer than the time over which E/N changes. In this assignment we will investigate how long and over what distance an electron swarm comes into equilibrium with a change in the electric field. By doing so, we will be able to assess when the LFA is a good approximation.

1. Assume that at t = 0 the electric field makes a step function change from $0 \rightarrow (E/N)_0$ and remains constant thereafter.



Integrate the electron energy conservation equation as a function of time to calculate the time that T_e takes to come into equilibrium with $(E/N)_0$. You may assume that the equilibration time, τ_{eq}, is the time when T_e reaches 90% of its asymptotic steady state value, T_{e0}. The conditions you should use for your calculations are:

- T_e (t = 0) = 0.05 eV,

- Use the Ideal Molecule cross sections including momentum transfer collisions (as approximated by elastic scattering), electronic excitation collisions and ionization collisions.
- Mass of ideal molecule = 28 AMU
- T_{gas} = 300 K
- Assume the electron density remains constant.
- Recall that the units of E/N = [Electric field (V/cm)] / [Gas Density (1/cm^3)] = V-cm^2. E/N is often expressed in units of Townsend (Td) where 1 Td = 10^{-17} V-cm^2.

Perform your calculations for

a) $(E/N)_0$ = 1, 10, and 100 Td at a gas pressure of 1 Torr.
b) Gas pressure = 0.1, 1 and 10 Torr for $(E/N)_0$ = 10 Td.

Please comment on your results and on the scaling of the equilibration time with $(E/N)_0$ and gas pressure.

HINT: You need to integrate the expression $\frac{\partial T_e}{\partial t} = \mathit{function}(T_e)$, as a function of time. This is a first order ordinary-differential-equation (ODE). This type of problem is called an initial value integration where you start with a known value of T_e at t = 0 and integrate (or march) forward in time, recording the value of T_e as time increases. For example, a very simple integration scheme is $T_e(t+\Delta t) = T_e(t) + \frac{\partial T_e(t)}{\partial t}\Delta t$. You will need to keep Δt small enough so that the fractional change in T_e during any timestep is small. You can also use ODE solvers in applications such as MATLAB as well. The integration in this assignment *cannot be done analytically or by an integration by parts.* It must be done *numerically*.

2. Is there any way to "collapse" the results from Problem 1 into a more convenient form using a scaling parameter or scaling law?

3. Over what distance does the electron swarm in Problem 1 move in the time required for T_e to come into equilibrium for:

a) E/N = 1 Td, gas pressure = 0.1 Torr
b) E/N = 100 Td, gas pressure = 100 Torr