In this homework assignment, we will predict the characteristics of a positive column glow discharge for realistic operating conditions. Use the following discharge and circuit conditions.

A cylindrical discharge tube (length $L$, diameter $d$) is connected in series with a DC power supply (voltage $V_0$) and ballast resistor ($R_B$ ohms). The gas pressure inside the tube is 1 Torr at a gas temperature of $T_{gas} = 300$ K.

Use the cross sections for the ideal molecule and $M = 20$ AMU. Assume that the only inelastic energy loss process is electron impact ionization, and that electron loss is dominated by ambipolar diffusion. You may assume that the ion temperature, $T_I$ is equal to the gas temperature, $T_{gas}$. For these particular conditions, the plasma properties are obtained from the following:

Electron Temperature: Obtained from the electron continuity equation:

$$ T_e: \ \frac{\partial n_e}{\partial t} = 0, \quad k_{ion}(T_e)N - \frac{D_s(T_e)}{\Lambda^2} = 0 $$

$(E/N)$: Obtained from the electron temperature equation:

$$ \frac{E}{N} : \quad T_e = T_g + \left( \frac{2}{3k_B} \right) \left( \frac{M}{2m_e} \right) \left( \frac{q^2}{m_e k_m(T_e)} \right) \left( \frac{E}{N_{gas}} \right)^2 - \sum_i \frac{\Delta \varepsilon_i n_i(T_e) N_{gas}}{N_{gas} k_m(T_e)} $$

Electron density: Obtained from the current density

$$ n_e : \quad j = \frac{I}{\pi \left( \frac{d}{2} \right)^2} = \sigma E = \frac{n_e q^2}{m_e k_m(T_e)} \left( \frac{E}{N_{gas}} \right) $$

Recall that the electron collision frequency for process $i$ is related to the rate coefficient $k_i$ by

$$ \nu_i = k_i N_{gas} \quad \text{and that the ambipolar diffusion coefficient is} \quad D_s(T_e) = D_s(T_I) \left( 1 + \frac{T_e}{T_I} \right) $$

where

- $T_e$: Electron temperature (eV)
- $k_{ion}(T_e)$: Rate coefficient for ionization (cm$^3$/s)
- $k_m(T_e)$: Rate coefficient for momentum transfer (cm$^3$/s)
k_i(T_e), \Delta \epsilon_i \quad \text{Rate coefficient for inelastic process } i \text{ with energy loss } \Delta \epsilon_i \text{ (cm}^3/\text{s, eV)}

D_a(T_e) \quad \text{Ambipolar diffusion coefficient (cm}^2/\text{s)}

D_i(T_i) \quad \text{Ion diffusion coefficient (cm}^2/\text{s)}

\frac{d}{2} \quad \text{Diffusion length for cylinder (cm)}

\Lambda = \frac{d^2}{2.405} \quad \text{Diffusion length for cylinder (cm)}

T_{\text{gas}} \quad \text{Gas temperature (K)}

T_I \quad \text{Ion temperature (K)}

k_B \quad \text{Boltzmann’s constant (1.6 x 10^{-12} erg/eV, 1.38 x 10^{-16} erg/K)}

N_{\text{gas}} \quad \text{Gas density (cm}^3)

M, m_e \quad \text{Gas atom mass, electron mass (g)}

q \quad \text{Elementary charge (c)}

j \quad \text{Current density (A/cm}^2)

I \quad \text{Current (A)}

\sigma \quad \text{Conductivity (1/Ohm-cm)}

(Recall that the gas density } N_{\text{gas}} = 9.654 \times 10^{18} P_{\text{gas}}/T_{\text{gas}} \text{ cm}^{-3}, \text{ when the gas pressure } P_{\text{gas}} \text{ is in Torr and the gas temperature } T_{\text{gas}} \text{ is in K})

1. Plot E/N_{\text{gas}}, T_e, I (total current), V_{\text{DIS}} \text{ (voltage across the discharge tube) and the electron density, } n_e, \text{ for the following parameters. Use } D_I = 500 \text{ cm}^2/\text{s} \text{ at a pressure of 1 Torr.}

   \text{L = 30 cm, } j = 15 \text{ mA/cm}^2, \quad 0.25 \text{ cm} < d < 2.5 \text{ cm}

Express E/N_{\text{gas}} \text{ in Townsend (1 Td = 10}^{17} \text{ V-cm}^2), T_e \text{ in eV, I in Amps and V_{\text{DIS}} \text{ in volts.}}

Discuss why each of E/N_{\text{gas}}, T_e, I, \text{ and } V_{\text{DIS}} \text{ change or does not change as the diameter of the discharge tube is changed. You may use values of rate coefficients you derived in previous homework assignments. You do not need to rederive the rate coefficients here.}

2. How will these values obtained in (1) change if } j \text{ is increased to 25 mA/cm}^2? \text{ Sketch your answers. (No additional numerical work is required.)}

3. How will the values in (1) change if } L \text{ is increased to 60 cm? Sketch your answers. (No additional numerical work is required.)}

4. Suppose we add an external source of ionization, S_{ion}, so that

\begin{equation}
\frac{\partial n_e}{\partial t} \approx 0 = n_e \left( k_{\text{ION}}(T_e)N - \frac{D_a(T_e)}{\Lambda^2} \right) + S_{ion}
\end{equation}

The external source could be, for example, photo-ionization from a UV lamp. \text{How will } T_e, n_e, \text{ E/N, V_{DIS} and I change as } S_{ion} \text{ increases? Assume } j \text{ remains a constant. (Explain your answer in words, no calculations are required.)}