EECS 517 / NERS 578 FALL 2012 HOMEWORK #3

The purpose of this assignment is to provide you with some insights for the orders of magnitudes of electron impact rate coefficients as a function of electron temperature, T_e; and how power is deposited in a glow discharge. You will find that at low T_e the electrons do not lose much energy per collision because they are not energetic enough to undergo inelastic collisions by exciting or ionizing the gas. As T_e increases and the rates of excitation and ionization increases, the energy loss-per-collision also increases.

Recall that a Maxwell Boltzmann electron energy distribution is for energy ε given by:

$$f(\varepsilon) = \frac{2}{\pi^{1/2}(k_B T_e)^{3/2}} \varepsilon^{1/2} \exp\left(\frac{-\varepsilon}{k_B T_e}\right) eV^{-1}$$

where k_B is Boltzmann's constant. Rate coefficients, k, for electron impact cross section $\sigma(\varepsilon)$ and electron mass m_e are obtained from:

$$k(T_e) = \int_0^\infty f(\varepsilon)\left(\frac{2\varepsilon}{m_e}\right)^{1/2} \sigma(\varepsilon) d\varepsilon \quad cm^3 s^{-1}$$

1. Use the cross sections for the "ideal molecule" (see below) and plot the electron impact rate coefficients for the processes listed below as a function of electron temperature, T_e. Use a Maxwell-Boltzmann for the electron energy distribution function. Plot the rate coefficients, $k(T_e)$, for 0.1 eV ≤ T_e ≤ 10 eV. Comment on the results and discuss why $k(T_e)$ has the shape it does for each process. (All rate coefficients can be obtained analytically. Please obtain the analytic solution. There is "physics" in the form of the analytic solution.)

a) Momentum transfer (use the elastic cross section)
b) Vibrational excitation
c) Ionization

NOTES:

a. Please display rate coefficients with units of cm^3/s.
b. Use log-scales for the axes of your plots (as appropriate) to show the full dynamic ranges of variables and computed quantities. Also plot only "reasonable" ranges of the computed values. For example, a range of at most 10^6 from the maximum value is "reasonable"…a range of 10^{30} is not reasonable. For example, if you have a rate coefficient of $k = 10^{-30}$ cm^3/s and a gas density of 1 Torr (about $N = 3 \times 10^{16}$ cm^{-3}), then the rate of collisions is $\nu = kN = 3 \times 10^{-14}$ s^{-1} which is about once every 1 million years. This is a "zero" rate of collisions!

c. Please show your units analyses and intermediate steps with a "nice narrative" which explains your logic.)

2. Define $P_j(T_e)$ as the "power loss / collision" for process j. This is the average rate at which an individual electron loses energy due in collision process j with an individual atom.

$$P_j(T_e) = \int_0^\infty f(\varepsilon)\left(\frac{2\varepsilon}{m_e}\right)^{1/2} \sigma_j(\varepsilon)\Delta\varepsilon_j \, d\varepsilon \quad eV - cm^3/s$$

where $\Delta\varepsilon_j$ is the energy loss by the electron in collision j. The total power deposition in the plasma (per unit volume) resulting from electron collisions is then

$$P_T(T_e) = n_e N_g \sum_j P_j(T_e) = n_e N_g \sum_j \int_0^\infty f(\varepsilon)\left(\frac{2\varepsilon}{m_e}\right)^{1/2} \sigma_j(\varepsilon)\Delta\varepsilon_j \, d\varepsilon \quad W/cm^3$$

where n_e is the electron density and N_g is the gas density. The sum is over all the collisional processes (which for this problem are momentum transfer, vibrational excitation and ionization.) Plot $P_j(T_e)$ as a function of T_e for each individual collisional process (momentum transfer, vibrational excitation and ionization) for $0.1 \text{ eV} \le T_e \le 10 \text{ eV}$. Assume the gas has molecular weight M = 28 AMU (as does N_2) and a gas temperature of 300 K. Please comment on the results. Discuss the physics that is responsible for the trends for each process and discuss how power deposition is divided among the three processes as a function of T_e.

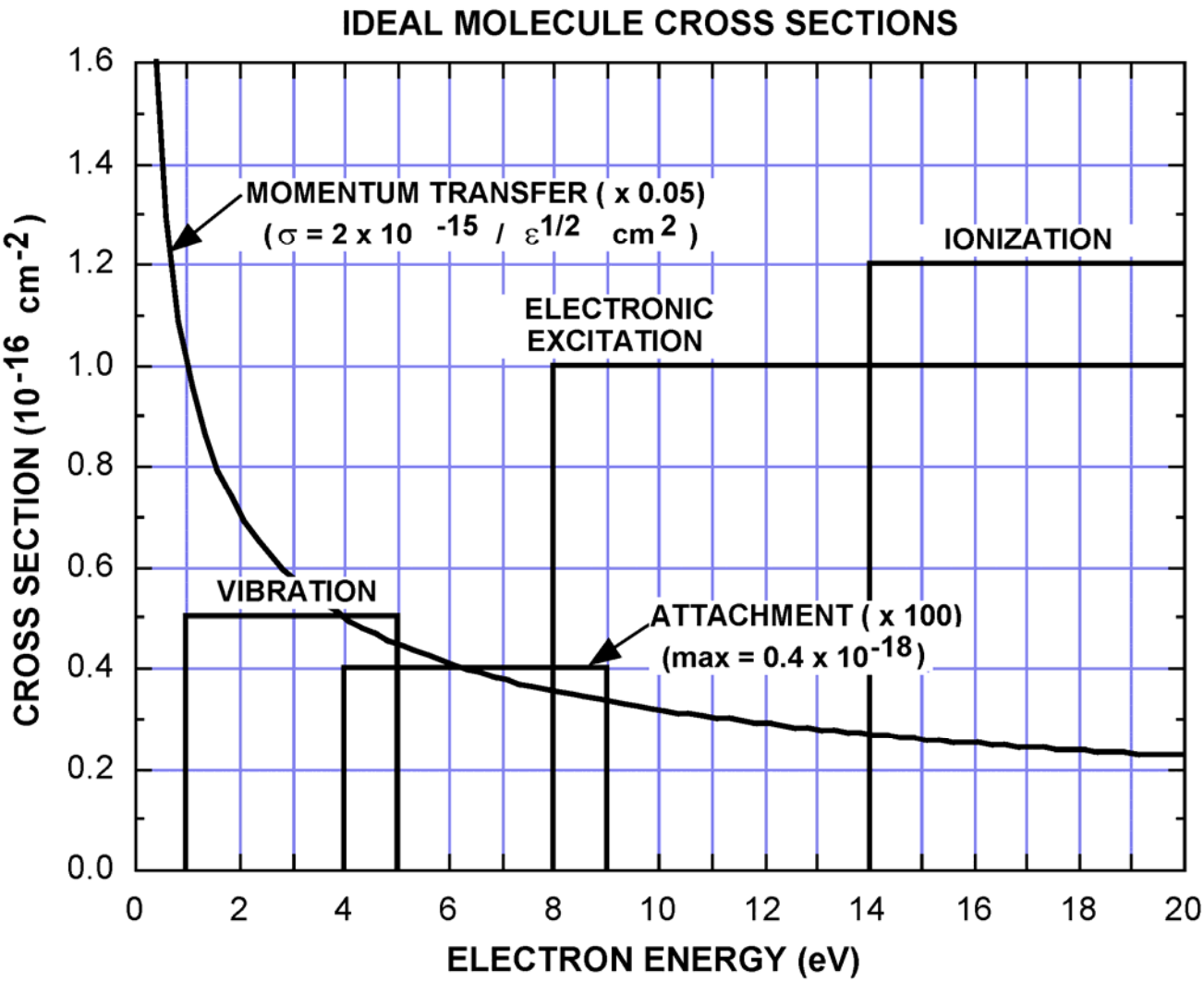
NOTE: The energy loss in an elastic collision with a gas atom having mass M by an electron with energy ε is $\left(\frac{2m_e}{M} \bullet \varepsilon\right)$. You can assume that the energy loss per collision for inelastic processes is the threshold energy.)

HINT: To obtain $P_j(T_e)$ for vibrational excitation and ionization, you do not need to do any more integrals that you have already done. You already have most of the answer from Problem 1.

3. Plot the total power deposition $P_T(T_e)$ [W/cm^3] for the conditions of Problem 2 for a gas pressure of P = 5 Torr, T_g = 300 K and $n_e = 10^{10}$ cm^{-3}. Please comment on the results. A typical CO_2 gas discharge laser is sustained in a tube that is 0.75 inch in diameter and 3 feet long operates at about 5 Torr and dissipates about 300 W of power (water cooled tube). The self sustaining electron temperature is about 2 eV. Estimate the electron density assuming the plasma is sustained in the ideal molecule.

4. Assume that vibrational excitation of the ideal molecule results in an attachment (that is, the loss of an electron by forming a negative ion). In the steady state (ignoring other electron sources and losses) we must have $\frac{\partial n_e}{\partial t} = 0$. What is the steady state electron temperature for a plasma sustained in a gas of ideal molecules for these conditions (that is, for the 3 processes of elastic collisions, vibrational excitation that results in an attachment and ionization)? Why?

HINT: DO NOT DO ANY ADDITIONAL CALCULATIONS FOR PROBLEM 4. YOU ALREADY HAVE THE ANSWER!



Notes:

1. For this problem, assume that the elastic cross section is the same as the momentum transfer cross section.

2. The momentum transfer cross section is $\sigma_m(\varepsilon) = \frac{2\times 10^{-15}}{\varepsilon^{1/2}} cm^2 eV^{1/2}$, where the electron energy ε is in eV. This means that if the electron energy is ε = 16 eV, then
$$\sigma_m(\varepsilon) = \frac{2\times 10^{-15}}{(16\,eV)^{1/2}} cm^2 eV^{1/2} = \frac{2\times 10^{-15}}{4\,eV^{1/2}} cm^2 eV^{1/2} = 0.5\times 10^{-15} cm^2$$

3. The electronic excitation and ionization cross sections are constant from their threshold energies to higher energies.

4. The vibration and attachment cross sections are "hat functions" extending only over the range indicated.

5. The maximum value of the attachment cross section is 0.4 x 10^{-18} cm^2.