In this homework assignment, we will calculate the characteristics of a positive column glow discharge for realistic operating conditions. Use the following discharge circuit conditions:

A cylindrical discharge tube (length \(l\), diameter \(d\)) is connected in series with a DC power supply (voltage \(V\)) and series ballast resistor (\(R_B\) ohms). The gas pressure is 1 Torr at a gas temperature of 300 K (\(N = 9.654 \times 10^{18} \frac{P_{\text{Torr}}}{T_K} \text{ cm}^{-3}\)).

Use the cross sections for the ideal molecule and \(M = 20\) AMU. Assume that the only inelastic energy loss process is electron impact ionization, and that electron loss is dominated by ambipolar diffusion. You may assume that the ion temperature, \(T_i\) is equal to the gas temperature. For these particular conditions, the plasma properties are obtained from the following:

**Electron Temperature:** Obtained from the electron continuity equation:
\[
T_e: \quad \frac{\partial n_e}{\partial t} = 0, \quad k_{\text{ion}}(T_e)N - \frac{D_A(T_e)}{\Lambda^2} = 0
\]

**(E/N):** Obtained from the electron temperature equation:
\[
\frac{E}{N} : \quad T_e = T_g + \left( \frac{2}{3k_B} \right) \left( \frac{m_e}{2k_m} \right) \left( \frac{q^2}{m_e k_m (T_e)} \right) \left( \frac{E}{N} \right)^2 - \sum_i \frac{\Delta \epsilon_i k_i(T_e)N}{N k_m(T_e)}
\]

**Electron density:** Obtained from the current density
\[
n_e : \quad j = \frac{I}{\pi \left( \frac{d}{2} \right)^2} = \sigma E = \frac{n_e q^2}{m_e k_m (T_e)} \left( \frac{E}{N} \right)
\]

Recall that collision frequencies are related to rate coefficients by \(v_i = k_i N\) and that the ambipolar diffusion coefficient is
\[
D_A(T_e) = D_i(T_i) \left( 1 + \frac{T_e}{T_i} \right)
\]

where
- \(T_e\) Electron temperature (eV)
- \(k_{\text{ion}}(T_e)\) Rate coefficient for ionization (cm\(^3\)/s)
- \(k_m(T_e)\) Rate coefficient for momentum transfer (cm\(^3\)/s)
- \(k_i(T_e), \Delta \epsilon_i\) Rate coefficient for inelastic process \(i\) with energy loss \(\Delta \epsilon_i\) (cm\(^3\)/s, eV)
1. Plot $E/N$, $T_e$, $I$ (total current), $V_{DIS}$ (voltage across the discharge tube) and $n_e$ for the following parameters. Use $D_I = 500 \text{ cm}^2 \text{s}^{-1}$ at a pressure of 1 Torr.

$L = 30 \text{ cm}, \quad j = 15 \text{ mA/cm}^2, \quad 0.25 \text{ cm} < d < 2.5 \text{ cm}$

Discuss why each of $E/N$, $T_e$, $I$, and $V_{DIS}$ change or does not change as the diameter of the discharge tube is changed. You may use values of rate coefficients you derived in previous homework assignments. You do not need to rederive the rate coefficients here.

2. How will these values obtained in (1) change if $j$ is increased to $25 \text{ mA/cm}^2$? Sketch your answers. (No additional numerical work is required.)

3. How will the values in (1) change if $L$ is increased to 60 cm? Sketch your answers. (No additional numerical work is required.)

4. Suppose we add an external source of ionization, $S_{ion}$, so that

$$\frac{\partial n_e}{\partial t} \approx 0 = n_e \left( k_{ION}(T_e) N - \frac{D_a(T_e)}{\Lambda^2} \right) + S_{ion}$$

The external source could be, for example, photo-ionization. How will $T_e$, $n_e$, $E/N$, $V_{DIS}$ and $I$ change as $S$ increases? Assume $j$ remains a constant. (Explain your answer in words, no calculations are required.)