In this homework assignment, we investigate the angular deflection of an electron during a collision. We found that the angular distribution of the scattered electron during a collision can have measurable effect on its rate of momentum transfer and so the circuit-properties (e.g., current and voltage) of a gas discharge.

Assume:

\[
f(\epsilon)d\epsilon = \frac{2}{\pi^{1/2}(k_BT_e)^{3/2}} \epsilon^{1/2} \exp\left(-\frac{\epsilon}{k_BT_e}\right)d\epsilon
\]

\[
\sigma(\epsilon) = \frac{10^{-15}}{\epsilon eV} \text{cm}^2 = 2\pi \int_0^\pi \sigma_0(\epsilon, \theta) \sin(\theta)d\theta
\]

\[
\sigma_0(\epsilon, \theta) = \frac{A(\epsilon)}{\epsilon} \cos^2\left(\frac{\theta}{2}\right)
\]

where

- \( \epsilon \equiv \) electron energy
- \( f(\epsilon) \equiv \) Electron energy distribution function. Here we assume that \( f(\epsilon) \) is a Maxwell-Boltzmann distribution.
- \( \sigma(\epsilon) \equiv \) Total electron collision cross section. (Note, \( \sigma(\epsilon = 1 \text{ eV}) = 10^{-15} \text{ cm}^2 \), \( \sigma(\epsilon = 10 \text{ eV}) = 10^{-16} \text{ cm}^2 \))
- \( \sigma_0(\epsilon, \theta) \equiv \) Differential cross section. There is no explicit dependence of the differential cross section on the azimuthal (or \( \phi \) angle). The \( \phi \) dependence has been integrated out and that value is contained in the normalization constant.
- \( A(\epsilon) \equiv \) Normalization constant
- \( T_e \equiv \) Electron temperature
- \( k_B \equiv \) Boltzmann’s constant (1.38 \( \times \) 10\(^{-16} \) erg/K, 1.6 \( \times \) 10\(^{-12} \) erg/eV, 1.6 \( \times \) 10\(^{-19} \) J/K)
- \( \theta \equiv \) Angular deflection in the polar direction of an electron following the collision (measured with respect to its velocity prior to the collision)

1. Plot \( \sigma_0(\epsilon, \theta) \) for \( \epsilon = 3 \text{ eV} \) as a function of \( \theta \) from (0,\( \pi \)) with \( n = 0, 1, 3, 10 \). Comment on the significance of \( n \). Why do we plot the cross section only from (0,\( \pi \)) and not (0,2\( \pi \))?

2. Plot the relative number of electrons scattering into the solid angle centered on \( \theta \) from (0,\( \pi \)) with \( n = 0, 1, 3, 10 \), and \( \epsilon = 3 \text{ eV} \). Comment on the results. Into what angle is the largest number of electrons scattered? Why?

3. Plot the electron momentum transfer collision frequency \( \nu_m \) for a gas pressure of \( P = 5 \text{ Torr} \), gas temperature \( T_g = 300 \text{ K} \), and over the range of 0 < \( T_e < 10 \text{ eV} \) with \( n = 0, 1, 3, 10 \). Comment on the results and the significance of \( n \).
HINTS: a. Solve for $A(\varepsilon)$ from $\sigma_T(\varepsilon) = 2\pi \int_0^\varepsilon \sigma_0(\varepsilon, \theta) \sin(\theta) d\theta$

b. Use $\frac{V_m}{N} = k_m = \int_0^\varepsilon f(\varepsilon) \left( \frac{2\varepsilon}{m_\varepsilon} \right)^{1/2} \sigma_m(\varepsilon) d\varepsilon$ where the momentum transfer cross section is

$\sigma_m(\varepsilon) = 2\pi \int_0^\pi \sigma_0(\varepsilon, \theta) (1 - \cos(\theta)) \sin(\theta) d\theta$

4. If the current density, $j$, \( j = qn_e \mu_e E \), \( \mu_e = \frac{|q|}{m_e V_m} \) and electron density, $n_e$, are constant what happens to the electric field as $n$ increases from 0 to 10. Why?