RATE EQUATIONS

Rate equations are expressions of the time rate of change of the density of a species. For example, consider the time rate of change of the electronically excited state M^*.

		Rate coefficient
$e + M \rightarrow M^* + e$	Excitation	$k_1(T_e)$ cm^3/s
$e + M^* \rightarrow M + e$	Superelastic	$k_2(T_e)$ cm^3/s
$e + M^* \rightarrow M^+ + 2e$	Ionization	$k_3(T_e)$ cm^3/s
$M^* \rightarrow M + h\nu$	Radiative Relaxation	A $\frac{1}{s}$
$e + M^+ \rightarrow M^*$	Recombination	$k_4(T_e)$ cm^3/s

The rate equation for the density of M^* is

$$\frac{dM^*}{dt}\left(\frac{1}{cm^3\text{-}s}\right) = n_e k_1(T_e) M - n_e k_2(T_e) M^* - n_e k_3(T_e) M^* - M^* A + n_e k_4(T_e) M^+$$

where M, M^+ and n_e are the densities of the ground state, ion and electrons $\left(\frac{1}{cm^3}\right)$.

In the steady state, $\frac{dM^*}{dt} = 0$. If n_e, M and M^+ are known, one can solve for M^*

$$\frac{dM^*}{dt} = 0 \rightarrow M^*\left[n_e k_2 + n_e k_3 + A\right] = n_e k_4 M^+ + n_e k_1 M$$

$$M^* = \frac{n_e k_1(T_e) M + n_e k_4(T_e) M^+}{n_e k_2(T_e) + n_e k_3(T_e) + A}$$

In a rate-equation model of a plasma, you construct a separate rate equation for each species and temperature

$$\frac{dM}{dt} = -n_e k_1(T_e) M + \cdots$$

$$\frac{dM^*}{dt} = +n_e k_1(T_e) M + \cdots$$

$$\frac{dM^+}{dt} = +n_e k_3(T_e) M^* + \cdots$$

$$\frac{dT_e}{dt} = \cdots$$

Where the rate equations contain terms for all processes that change a density or temperature. These rate equations can then be simultaneously integrated as a function of time.