DERIVATION OF AMBIPOLAR DIFFUSION COEFFICIENT AND AMBIPOLAR ELECTRIC FIELD

Start with continuity equations for electrons and ions. This accounts for transport due only to diffusion processes. This would be, for example, for transport of electrons and ions to the walls in a narrow plasma tube whose length is much greater than the radius. The transport of electrons and ions along the axis of the plasma tube due to an externally applied electric field is not accounted for here. The ambipolar electric field included here is a self generated electric field to keep the flux of electrons and ions to the walls equal – it is not the applied electric field along the axis.

$$\frac{\partial n_e}{\partial t} = -\nabla \bullet \Gamma_e = -\nabla \bullet (-D_e \nabla n_e - \mu_e E_A n_e) = D_e \nabla^2 n_e + \mu_e E_A \nabla n_e + \mu_e n_e \nabla \cdot E_A$$

$$\frac{\partial N_I}{\partial t} = -\nabla \bullet \Gamma_I = -\nabla \bullet (-D_I \nabla N_I + \mu_I E_A N_I) = D_I \nabla^2 N_I - \mu_I E_A \nabla N_I - \mu_I N_I \nabla \cdot E_A$$

Where

E_A	Ambipolar electric field (V/cm)
n_e, N_I	Electron, ion density ($1/cm^3$)
Γ_e , Γ_I	Electron, ion flux ($1/cm^2$-s)
μ_e , μ_I	Electron, ion mobility (cm^2/V-s)
D_e , D_I	Electron, ion free diffusion coefficient (cm^2/s)

Assuming that $\nabla \bullet E_A = 0$ then

$$\frac{\partial n_e}{\partial t} = D_e \nabla^2 n_e + \mu_e E_A \nabla n_e \qquad \text{(a)}$$

$$\frac{\partial N_I}{\partial t} = D_I \nabla^2 N_I - \mu_I E_A \nabla N_I \qquad \text{(b)}$$

Multiply (a) by μ_I, multiply (b) by μ_e, and add them together while assuming that $n_e = N_I$, $\frac{\partial n_e}{\partial t} = \frac{\partial N_I}{\partial t}$, $\nabla n_e = \nabla N_I$ and $\nabla^2 n_e = \nabla^2 N_I$ then

$$\mu_I, \frac{\partial n_e}{\partial t} = \mu_I, D_e \nabla^2 n_e + \mu_I, \mu_e E_A \nabla n_e$$

$$\mu_e \frac{\partial N_I}{\partial t} = \mu_e D_I \nabla^2 N_I - \mu_e \mu_I E_A \nabla N_I$$

One then obtains

$$\frac{\partial n_e}{\partial t} = \left[\frac{D_e \mu_I + D_I \mu_e}{\mu_e + \mu_I}\right] \nabla^2 n_e = D_A \nabla^2 n_e$$

where the ambipolar diffusion coefficient is:

$$D_A = D_I \frac{\left(1 + \frac{D_e}{D_I}\frac{\mu_I}{\mu_e}\right)}{\left(1 + \frac{\mu_I}{\mu_e}\right)}.$$

Using the Einstein relation $D = \frac{kT}{q}\mu$ and assuming that $\mu_e >> \mu_I$, then

$$D_A = D_I \frac{\left(1 + \frac{T_e}{T_I}\right)}{\left(1 + \frac{\mu_I}{\mu_I}\right)} \approx D_I \left(1 + \frac{T_e}{T_I}\right)$$

Both electrons and ions appear to diffusion with the same coefficient, D_A, which includes the effects of both their own free diffusion and the acceleration by the ambipolar electric field, E_A.

We can now solve for E_A. Starting with the expressions for fluxes, subtract Γ_e from Γ_I (noting that $\Gamma_e = \Gamma_I$), and solve for E_A.

$$\Gamma_e = -D_e \nabla n_e - \mu_e n_e E_A$$
$$\Gamma_I = -D_I \nabla N_I + \mu_I N_I E_A$$

$$E_A = -\frac{D_e \nabla n_e - D_I \nabla N_I}{\mu_e n_e + \mu_I N_I} \approx -\frac{\nabla n_e}{n_e}\frac{D_e - D_I}{\mu_e + \mu_I}$$

For $D_e >> D_I$, $\mu_e >> \mu_I$, $E_A \approx -\frac{\nabla n_e}{n_e}\frac{D_e}{\mu_e} = -\frac{\nabla n_e}{n_e}\frac{k_B T_e}{q}$

For $\nabla \sim \frac{1}{\Lambda}$ (Λ is the diffusion length), then $E_A \approx \frac{1}{\Lambda}\frac{k_B T_e}{q}$.